Effects of nonlinear absorption and third order dispersion on soliton propagation in optical fiber

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Abstract—In this work, we use the generalized nonlinear Schrödinger equation to study the propagation of ultrashort optical pulses in the presence of self-phase modulation, nonlinear absorption and third-order dispersion. The combined effect of the third-order dispersion and nonlinear absorption on amplitude, center location and phase of the soliton has been investigated by an approximate analytical method.

Solitons are a fundamental phenomenon in nonlinear dynamics and have attracted the attention of researchers from physical and mathematical sciences. Optical solitons have been the subject of intensive theoretical and experimental studies for many years. Solitons were studied in nonlinear optics, plasma physics, particle physics, biological systems and Bose-Einstein-condensation. In nonlinear optics, these special types of optical wave packets appearing as the result of interplay between dispersion and nonlinearity are of special interest because of their important applications in telecommunications [1-3] and optical data processing [4-5].

In most cases, the absorption is assumed to be linear. On the other hand, for special optical fibers, such as fiber optics and semiconductors doped lead-silica optical fibers [6-10], the influence of nonlinear absorption must be taken into account. The effects of nonlinear absorption on soliton propagation have been presented in [3]. However, the combined effect of the third-order dispersion and nonlinear absorption on soliton propagation in the optical fiber has not been adequately studied. It is the subject of our recent paper.

The nonlinear Schrödinger equation, which can be written as:

$$\frac{\partial A}{\partial z}+\frac{1}{v_g}\frac{\partial A}{\partial t}+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2}-i\gamma|A|^2A=-\frac{\alpha_0}{2}A-\frac{\alpha_2}{2}|A|^2A+\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}, \tag{1}$$

has been shown to govern the propagation of light in a optical fiber with linear and nonlinear absorption [1,3]. In this equation, A is the slowly varying envelope of an optical field along the optical fiber, β_2 and β_3 are the second and third order dispersion coefficient, v_g is the group velocity of the pulse and α_0, α_2 are the linear and nonlinear absorption coefficients respectively. The nonlinear parameter γ accounts for the third order nonlinear polarization and is determined by the expression $\gamma = k_0 n_2 / A_{\mathrm{eff}}$, where k_0 is the wavenumber, n_2 - the nonlinear refractive index and the parameter A_{eff} is known as the effective core area of the optical fiber.

Applying the transformation $t' = t - z/v_g$, Eq.(1) is re-written as follows:

$$\frac{\partial A}{\partial z}+\frac{i}{2}\beta_2\frac{\partial^2 A}{t'^2}-i\gamma|A|^2A=-\frac{\alpha_0}{2}A-\frac{\alpha_2}{2}|A|^2A+\frac{\beta_3}{6}\frac{\partial^3 A}{t'^3}. \tag{2}$$

In a dimensionless form with rescaled variables

$$Z=\frac{z}{L_D},\ T=\frac{t'}{T_0},\ u=\sqrt{\gamma L_D}A, \tag{3}$$

where:

$$L_D=\frac{T_0^2}{|\beta_2|},\ \delta_3=\frac{\beta_3}{6|\beta_2|T_0},\ \Gamma_1=\frac{\alpha_0}{2}L_D,\ \Gamma_2=\frac{\alpha_2}{2\gamma}, \tag{4}$$

the NLS equation reads

$$i\frac{\partial u}{\partial Z}+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2u=-i\Gamma_1 u-i\Gamma_2|u|^2u+i\delta_3\frac{\partial^3 u}{\partial T^3}. \tag{5}$$

Here we have chosen sgn(β_2) = -1, which corresponds to the anomalous GVD region, where bright solitons can exist. L_D is the dispersion length, T_0 is the width of the incident pulse.

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When the terms $-i\Gamma_1 u, -i\Gamma_2 |u|^2 u$ and $i\delta_3 \frac{\partial^3 u}{\partial T^3}$ are considered as small perturbations, then Eq. (2) is rewritten as:

$$i\frac{\partial u}{\partial Z}+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2 u=i\varepsilon(u), \tag{6}$$

where

$$\varepsilon(u)=-\Gamma_1 u-\Gamma_2 |u|^2 u+\delta_3 \frac{\partial^3 u}{\partial T^3} \tag{7}$$

is a small perturbation that can affect u, u^* and their derivatives. In the presence of a small perturbation, the solution of the NLS equation can be written as:

$$u(Z,T)=\eta(Z)\operatorname{sech}\left[\eta(Z)(T-q(Z))\right]\exp\left[i\sigma(Z)-i\kappa(Z)T\right].$$

Here $\eta(Z)$ is the normalized amplitude of the soliton, and $\kappa(Z)$ represents the frequency of the soliton, $\sigma(Z)$ - the time-independent phase shift., $q(Z)$ is the location of the soliton center. According to the perturbation method [1-3], the evolution of soliton parameters is determined according to the following equations:

$$\frac{d\eta}{dZ}=-2\Gamma_1\eta-\frac{4}{3}\Gamma_2\eta^3, \tag{8}$$

$$\frac{d\kappa}{dZ}=0, \tag{9}$$

$$\frac{dq}{dZ}=-\kappa+\eta^2\delta_3+3\kappa^2\delta_3, \tag{10}$$

$$\frac{d\sigma}{dZ}=\frac{1}{2}\left(\eta^2-\kappa^2\right)+\left(3\kappa\eta^2+\kappa^3\right)\delta_3. \tag{11}$$

The solutions of the above equations are:

$$\eta(Z)=\eta_0\left[1+\frac{2}{3}\frac{\Gamma_2}{\Gamma_1}\eta_0^2\left[1-\exp(-4\Gamma_1 Z)\right]\right]^{-1/2}\exp(-2\Gamma_1 Z), \tag{12}$$

$$\kappa(Z)=\kappa_0, \tag{13}$$

$$q(Z)=-\kappa_0 Z+q_0+ +\left\{3\kappa_0^2 Z+\frac{3}{8\Gamma_2}\ln\left[1+\frac{2}{3}\frac{\Gamma_2}{\Gamma_1}\eta_0^2\left[1-\exp(-4\Gamma_1 Z)\right]\right]\right\}\delta_3, \tag{14}$$

$$\sigma(Z)=\frac{3}{16\Gamma_2}\ln\left[1+\frac{2}{3}\frac{\Gamma_2}{\Gamma_1}\eta_0^2\left[1-\exp(-4\Gamma_1 Z)\right]\right]-\frac{1}{2}\kappa_0^2 Z +\left\{\frac{9}{8\Gamma_2}\ln\left[1+\frac{2}{3}\frac{\Gamma_2}{\Gamma_1}\eta_0^2\left[1-\exp(-4\Gamma_1 Z)\right]\right]-\kappa_0^3 Z\right\}\delta_3+\sigma_0, \tag{15}$$

where $\eta_0, \kappa_0, q_0, \sigma_0$ are the initial values of soliton amplitude, velocity, center location and time-independent phase, respectively.

From Eqs. (14-15), we see that the third-order dispersion only affects the phase and position of the soliton. Figure 1 shows the center location of the soliton as a function of the normalized propagation distance Z for various values of δ_3. From this figure we see that the magnitude of the parameter q is proportional to the magnitude of the third-order dispersion coefficient.

However, in the case of the positive δ_3 (blue line), the location of the soliton center is moved to the positive of the axis. When δ_3 is negative, it is moved in the opposite direction. These results coincide with the results obtained by numerical methods in [1, 11].

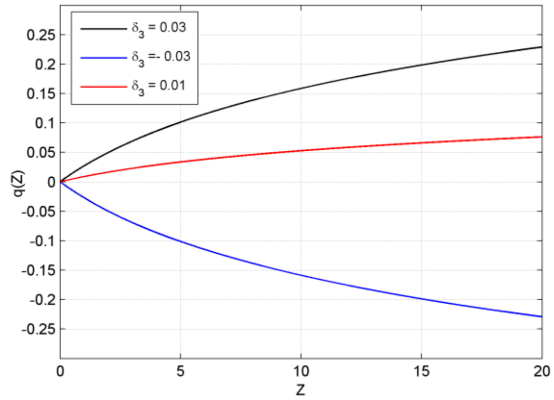

Fig. 1. The center location of the soliton as a function of the normalized propagation distance Z for various values of δ_3. The initial condition u(T,0) = sechT and the parameters used are $\eta_0=1$ $\Gamma_1=0.02$.

The influence of nonlinear absorption on the parameter $q(Z)$ is shown in Fig. 2. We see that the magnitude of the parameter $q(Z)$ is inversely proportional to the nonlinear absorption coefficient. So, in the case of the positive δ_3, the influence of third-order dispersion and nonlinear absorption on the parameter q is opposite

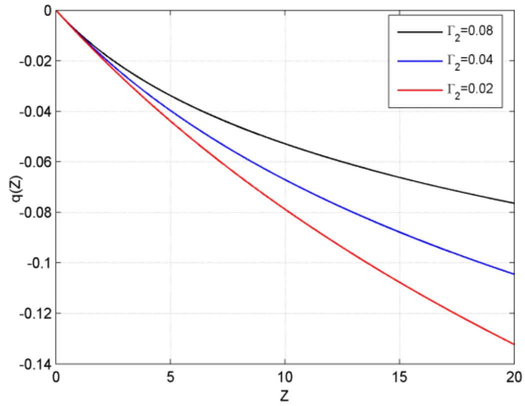

Fig. 2. The center location of the soliton as a function of the normalized propagation distance Z for various values of Γ_2. The initial condition u(T,0) = sechT and the parameters used are $\eta_0=1$ $\Gamma_1=0.02$.

In the presence of fiber loss, pulse amplitude decreases when the propagation distance Z increases [see Eq. (12)]. One can avoid this limitation by compensating fiber loss by means of continuous amplification. In this case, Eq. (8) is rewritten as:

$$\frac{d\eta}{dZ} = 2G\eta - 2\Gamma_1\eta - \frac{4}{3}\Gamma_2\eta^3, \qquad (16)$$

where $G = gL_D / 2$, with g being the gain per unit length. Thus the amplitude of the soliton is constant if $d\eta / dZ = 0$, that is, the amplification factor must satisfy the following condition:

$$G = \Gamma_1 + 2\Gamma_2\eta^2 / 3,$$

in fact, the amplification factor is usually chosen to satisfy the condition $G = \Gamma_1 + 2\Gamma_2\eta_0^2 / 3$ with η_0 being the initial value of soliton amplitude.

In this work we have investigated the propagation of optical solitons in the presence of both nonlinear absorption and third-order dispersion. We have derived the analytical formulas of soliton parameters describing soliton propagation in optical fibers. Based on these results, the combined effect of third-order dispersion and nonlinear absorption on amplitude, center location and phase of the soliton has been adequately studied.

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