Why phenomenological theory cannot be used to analyze the dynamics of dark photorefractive solitons

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Abstract—The currently used theory of bright and dark spatial photorefractive (PR) solitons is phenomenological. In this work, we consider the dynamics of dark beams using a microscopic model based on the PR transport equations. It is generally believed that such a theory is more accurate. It has been found that these two approaches can give completely different predictions regarding the formation time of dark beams. The discrepancies can reach up to two orders of magnitude. An approximate analytical solution was presented, and a new time constant was introduced, considering crystals from three classes of PR materials: ferroelectrics, sillenites, and semiconductors.

Spatial screening solitons, i.e., self-trapping beams that do not diffract during propagation, are one of the most interesting nonlinear effects produced in photorefractive materials. The simplest solitons (1+1)D can be created as bright and dark; in the latter case, it means a black notch is superimposed on an otherwise uniform background illumination, see Fig.1. It is generally believed that the theory of these beams is well established, its description can be found in articles and monographs [1–2]. The soliton beam analysis combines the paraxial wave equation with a system of microscopic material equations used to determine the distribution of the electric field, which induces a local change in the refractive index through the EO effect. However, what is commonly overlooked is that the standard theory imposes such strong constraints on the material equations that the result is a phenomenological (macroscopic) theory. It turns out that such an approximate theory correctly reproduces the properties of bright solitons [3], which was also confirmed experimentally [2, 4], but, as will be shown in this article, it differs significantly from the more accurate microscopic theory when it comes to the dynamics of dark solitons. To the author's knowledge, the correctness of the macroscopic approach has never been verified numerically or experimentally. It was automatically assumed that a method that works correctly for bright beams will also be suitable for dark ones. This is not true, and the discrepancies between the results of microscopic and macroscopic theories can reach almost two orders of magnitude. This article shows how to formulate a corrected microscopic dynamics equation and obtain a simple, approximate, universal solution that agrees well with numerical results. The dark beam parameters are shown in Fig. 1.

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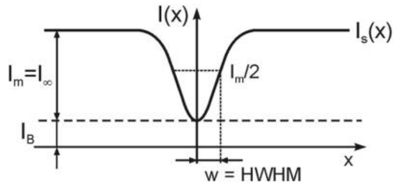

Fig. 1. Light intensity distribution for a Gaussian beam with half-width (HWHM) for a dark beam, where $I_s(x)$ is signal beam intensity distribution, and IB is background light intensity.

A beam is assumed to be a Gaussian beam with the intensity given by:

$$I(x) = I_s(x) + I_B = I_0\left[1 - m\exp\left(-\ln 2\cdot x^2/w^2\right)\right], \quad (1)$$

where $I_0 = I_\infty + I_B$ and w is the width at a half maximum (HWHM); for dark beams, the contrast beam coefficient $m = I_\infty/(I_B + I_\infty)$ can be written as $m = \delta/(1 + \delta)$, where $\delta = I_\infty/I_B$ is commonly used for the parametrization of dark solitons.

The one-carrier band transport model

The photorefractive standard transition model assumes one dopant level (donors with density N_D) and one compensatory level (acceptors with density N_A). The rate equations for electrons can be written as

$$\partial N_D^+/\partial t = S(I_s + I_B)\left(N_D - N_D^+\right) - \gamma n N_D^+ \quad (2a)$$

$$\partial(N_D^+ - n)/\partial t = -\mu\cdot\nabla(nE) = -\mu E_a \nabla(n\cdot e) \quad (2b)$$

$$N_D^+(x,t) = N_A\left(1 + L_A\nabla e(x,t)\right), \quad (2c)$$

where $e(x,t) = E(x,t)/E_a$ denotes the normalized electric field and $L_A = \varepsilon_0\varepsilon_r E_a/(qN_A)$ - the characteristic length, S - photoionization cross-section [m^2/J], n - electron density, μ - electron mobility and γ - recombination coefficient. In the case of low-intensity beams, we have n "N_D^+, $N_D^+ - N_A$.

Standard theory as a macroscopic approximation

We aim to find a temporal equation for forming the space-electric field when the optical beam is abruptly switched on $I(x,t) = I(x)\theta(t)$, where $\theta(t)$ is the Heaviside step function. In the standard analysis of screening solitons, two strong assumptions are made: (1) $L_A\nabla e << 1$ and (2) $\partial N_D^+/\partial t = 0$ in set Eq. (2a) but not in Eq. (2b) [1, 2, 4]. In literature, the last statement is tried to be justified physically, but it is obviously inconsistent mathematically. Thus, the equation for the concentration of excited electrons is written as:

$$n(x,t) \approx SI(x)\theta(t)(N_D - N_A)\tau_r, \tag{3}$$

where $\tau_r = 1/(\gamma N_A)$ denotes the electron recombination time. Equation (3) reads that the distribution of free carriers is established immediately after the appearance of the optical beam. Combining Eq. (3) with (2a) and (2c), we arrive at the equation :

$$\partial e(x,t)/\partial t + u(x)e(x,t)/\tau_{die} = 1/\tau_{die} \tag{4}$$

with the normalized light intensity distribution given by $u(x) = [I_s(x) + I_B]/(I_\infty + I_B)$. τ_{die} is the dielectric relaxation time connected to the background illumination, i.e., $\tau_{die} = \varepsilon_0\varepsilon_r/(q\mu n_\infty)$, with $n_\infty = n(x\to\infty)$. Equation (4) is commonly used for the temporal evolution description of the electric field in PR crystals for both bright and dark optical beams [1–5]. The solution of Eq. (4) is:

$$e_{stand}(x,t) = 1/u(x) + [1 - 1/u(x)]\exp(-t/\tau_{stand}). \tag{5}$$

According to Eq. (5), the electric field varies monotonously to the stationary state $e(x,\infty) = 1/u(x)$ with the standard response time $\tau_{stand} = \tau_{die}/u(x)$. The essential point is that the presented scheme is, in fact, a phenomenological approach. Under the approximation (3), Eqs. (2a)-(2c) can be rewritten in terms of photoconductivity σ and charge density ρ, as: $\sigma(x) = C\cdot[I_s(x) + I_B]$, where $C = \sigma_\infty/(I_\infty + I_B)$, i.e., material photoconductivity is proportional to the light intensity, $\partial\rho/\partial t = -\nabla\mathbf{J} = -\nabla(\sigma\mathbf{E})$ and $\nabla\mathbf{E} = \rho/\varepsilon$ (Gauss law). It can be shown that these equations lead directly to Eq. (5). The macroscopic approach is simple and universal but approximate because it ignores microscopic parameters specific to different materials.

The microscopic time evolution equation of the electric field

It turns out that Eq. (4), resulting from the approximation (3), completely fails to consider the dynamics for dark beams. We will formulate a dynamic equation consistent with the model described by Eqs. (2a)–(2c). Note that if we put $\partial N_D^+/\partial t = 0$ in Eq.(2), then Eq. (2b) implies $\partial n/\partial t = 0$ and $\nabla\mathbf{J} = 0$. Following the physics of the charge transport process for dark beams, we can not assume $\partial n/\partial t = 0$, but we assume that the changes in the current density are small, i.e. $\nabla\mathbf{J} \sim 0$. This is equivalent to the condition:

$$n(x,t)\cdot E(x,t) \approx n_\infty E_a\,. \tag{6}$$

This is the crucial equation for the further analysis. By inserting (6) into the system (2a)–(2c) after some algebra, one obtains the looked-for dynamic equation :

$$L_A\tau_{S\infty}\nabla(\partial e/\partial t) = u(x) - (1 + L_A\nabla e)/e\,, \tag{7a}$$

where $\tau_{S\infty} = r/(SI_0)$ with $r = N_A/N_D$. (7b)

Stationary state

As seen from Eq. (7a), the electric field distribution in a steady state regime is determined by the equation [3]:

$$L_A\nabla e(x) = u(x)e(x) - 1\,. \tag{8}$$

The profiles $e(x)$ calculated from Eq. (8) and the standard expression $1/u(x)$ can be significantly different, as shown in Fig. 2, taking into account the SBN crystal.

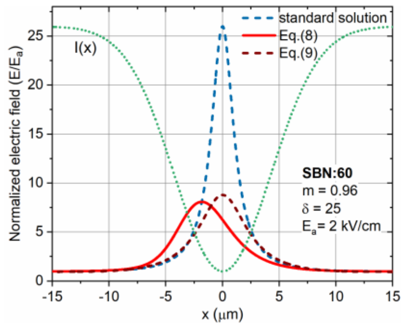

Fig. 2. Spatial distributions of the internal normalized electric field formed in SBN crystals by a dark optical Gaussian beam with FWHM =10 μm. For a given value $\delta = 25$, we find a large discrepancy between the standard ($1/u(x)$) and microscopic solution – Eq. (8).

Omitting the spatial shift, the analytical expression for the steady-state electric field distribution that correctly approximates the numerical solution may be written in the form:

$$e_{approx} \approx 2(\rho+1)\left[\frac{1}{I_{norm}(x)+\alpha} - \frac{1}{\rho+\alpha}\right] + 1\,, \tag{9}$$

where $I_{norm}(x) = I_s(x)/I_B$ denotes the normalized beam intensity, $\alpha = M(1 + M/10)$, $M = \max[1/u(x)]/\max[e(x)]$. Equation (9) is a counterpart of the phenomenological approximation $e_{stand} = 1/u(x)$, where $L_A = 0$ is assumed.

Temporal evolution and microscopic time constant

The time evolution Eq. (7a) does not have a closed analytical solution. However, we can get an approximate analytical solution as follows. It is known that in dark regions of the light distribution, the space-charge field increases quasi-exponential. Therefore, an approximate solution to Eq. (7) in analogy to Eq. (5) can be postulated as:

$$e(x,t) \approx e_\infty(x) + [1 - e_\infty(x)]\exp(-t/\tau_{sc}), \tag{10}$$

where $e_\infty(x) = e(x, t\to\infty)$, $e_{0,\infty} = e(x_0, t\to\infty)$.

To determine the total space-charge formation time, the point $x_0 = 0$ (center of an optical beam) is taken, and the time constant can be presented as:

$$\tau_{sc} \approx (1/2)\tau_{S\infty}\left(1+D_0\right)\left(1+D_0/u_0\right)/\left(1-u_0\right), \qquad (11)$$

where $\tau_{S\infty}$ is given by Eq. (7b) and $D_0 = u_0 e_{0,\infty}$. In the standard theory, the microscopic time constant replaces the macroscopic time constant $\tau_{stand} = \tau_{die}/u(x)$. The differences between τ_{sc} and τ_{stand} can be unexpectedly significant. Figure 3 compares the values of both constants considering materials from three classes of PR crystals: SBN:60 (strontium-barium-niobate) - ferroelectric (large static dielectric constant $\varepsilon_r \sim 1000$ and electro-optic (EO) coefficient $r_{33} \sim 100$ pm/V, the dielectric relaxation time $\tau_{die} \sim 1$s for the light intensity of 10 mW/cm^2), BSO (bismuth silicon oxide) – sillenite crystal with fast PR effect ($\varepsilon_r \sim 100$, with $r_{41} \sim 10$ pm/V and $\tau_{die} \sim 1$ ms) and GaAs:CrO_2 – semiconductor ($\varepsilon_r \sim 10$, the fastest material: $\tau_{die} \sim 1$ μs but with small EO coefficient $r_{41} \sim 1$ pm/V).

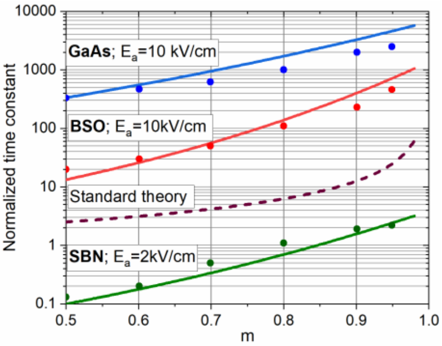

Fig. 3. Normalized time constant (τ/τ_{die}) vs. the modulation index m for a dark beam. The time constant according to Eq. (11) is compared with the standard theory ($\tau/\tau_{die} = 1/u(x{=}0)$, taking the material parameters for BSO, SBN:60, and GaAs:CrO_2 [3]. Points are the results of numerical calculations from the system of Eqs. (2a)–(2c).

It can be seen that the maximum differences between the numerical and approximate analytical results [Eqs. (10) and (11)] obtained for $t = \tau_{sc}$ reach 50%. Simultaneously, discrepancies in the predictions between the microscopic and macroscopic theory may attain two orders of magnitude.

Temporal wave equation for dark solitons

Knowing the time dependence of the charge-space field given by Eq. (10), we can write the paraxial wave equation as an explicit function of time. Traditionally, this equation is presented using the normalized complex amplitude $\phi = (C/I_B)^{1/2}\Phi$ and dimensionless coordinates $\xi = z/(kX_0^2)$, $\chi = x/X_0$, where $k = 2\pi/\lambda$, X_0 is the scale parameter which can be chosen arbitrarily. In a stationary state, the microscopic theory predicts a dark soliton profile similar to the standard theory. Therefore, we can apply Eq. (5) in practice. However, it is necessary to replace the standard time constant with the time constant (11). The resulting equation has the form [1, 5–7]:

$$i\frac{\partial\phi}{\partial\xi} + \frac{1}{2}\frac{\partial^2\phi}{\partial\chi^2} - \frac{B\cdot E_a}{|\phi|^2+1}\left[\delta + 1 + \left(|\phi|^2 - \delta\right)e^{-t/\tau sc}\right] = 0 \quad (12)$$

where $B = (1/2)(kX_0)^2 n_b^4 r_{eff}$.

Equation (12) yields the correct approximate dark soliton profile and formation time. This equation can be solved numerically by conventional beam propagation methods presented in other works [1, 5–7].

The article presents a novel description of the dynamics of (1+1)D dark beams, particularly dark screening solitons in PR materials consistent with the microscopic transport model. It was found that compared to the commonly used macroscopic theory, the differences in the results of both theories may be substantial. Experimental validation is still needed to confirm the results that were obtained.

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