Depolarization of circularly polarized light in birefringent crystal

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Received June 17, 2009; accepted June 29, 2009; published June 30, 2009

Abstract—In the paper we consider the depolarization phenomenon of light with partial temporal coherence and circular state of polarization passing through a birefringent medium. In order to describe the degree of polarization changes of light passing through a birefringent crystal we use the Mueller Stokes matrix formalism extended by an additional depolarization matrix. The results of experimental tests of theoretical predictions are presented here as well.

The degree of polarization (DOP) of partially coherent light diminishes during its transmission through birefringent media. The phenomenon is well known in solid anisotropic crystals such as quartz or lithium niobate [1,2], liquid crystals [3,4] as well as in high birefringent fibers used in polarimetric sensors [5] and telecommunication lines [6,7], and also in photonic liquid crystal fibers [8].

In our previous papers [2,3] we were dealing with the depolarization of light with a linear state of polarization. In this research, we analyze the problem of DOP changes of partially temporal coherent light with a circular state of polarization during propagation through lithium niobate with well known birefringence and dimensions. In general, both components of an electric field vector coupled into a birefringent medium may have different phase velocities, which results in an increase in the phase shift and state of polarization (SOP) changes. In addition, for partially coherent light characterized by coherence length *ΔL,* the phase shift may be so high that the light outgoing from a birefringent medium may be almost totally depolarized due to the fact that both electric field components of light are shifted into adjacent wave packages.

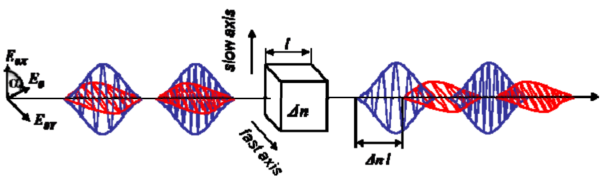

Fig. 1. Length shift of partially coherent light passing through an anisotropic medium with linear birefringence

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The depolarization phenomenon can be described by the Mueller-Stokes matrix formalism extended by depolarization matrix **[D$_c$]**:

$$\left[\mathbf{S}^{\mathbf{out}}\right]=\left[\mathbf{D}_{\mathrm{C}}\right]\cdot\left[\mathbf{M}\right]\cdot\left[\mathbf{S}^{\mathbf{in}}\right], \tag{1}$$

where [$\mathbf{S}^{\mathbf{in}}$] and [$\mathbf{S}^{\mathbf{out}}$] are input and output Stokes vectors respectively defined as:

$$[\mathbf{S}]=\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}=\begin{bmatrix} \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \\ \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle \\ \langle E_x E_y^* \rangle + \langle E_y E_x^* \rangle \\ i\left(\langle E_x E_y^* \rangle - \langle E_y E_x^* \rangle\right) \end{bmatrix} \tag{2}$$

[**M**] is the Mueller matrix of an anisotropic medium and is defined as:

$$[\mathbf{M}]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\alpha)+\sin^2(2\alpha)\cos(\delta) & (1-\cos(\delta))\sin(2\alpha)\cos(2\alpha) & \sin(2\alpha)\sin(\delta) \\ 0 & (1-\cos(\delta))\sin(2\alpha)\cos(2\alpha) & \sin^2(2\alpha)+\cos^2(2\alpha)\cos(\delta) & \cos(2\alpha)\sin(\delta) \\ 0 & \sin(2\alpha)\sin(\delta) & -\cos(2\alpha)\sin(\delta) & \cos(\delta) \end{bmatrix} \tag{3}$$

where θ is the azimuth of a fast axis, and $\delta = 2\pi\Delta n l/\lambda$ is the retardation of a medium.

[**D$_C$**] stands for a depolarization matrix defined as [2]:

$$[\mathbf{D}_{\mathrm{C}}]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_C & 0 & 0 \\ 0 & 0 & P_C & 0 \\ 0 & 0 & 0 & P_C \end{bmatrix} \tag{4}$$

where

$$P_C = P_L = \sqrt{1-\frac{4\left[1-\exp\left(-\dfrac{2\,|\,\Delta n\,|\,l}{\Delta L_L}\right)\right]}{\left(\dfrac{E_{0x}}{E_{0y}}+\dfrac{E_{0y}}{E_{0x}}\right)^2}} \tag{5}$$

for Lorentzian light sources, and

$$P_C = P_G = \sqrt{1 - \frac{4\left[1 - \exp\left(-2\left(\frac{|\Delta n|\, l}{\Delta L_G}\right)^2\right)\right]}{\left(\frac{E_{0x}}{E_{0y}} + \frac{E_{0y}}{E_{0x}}\right)^2}} \tag{6}$$

for Gaussian light sources.

As mentioned earlier, partially coherent light is being depolarized during propagation through a birefringent medium. DOP at the output of a medium can be calculated according to the formula [9]:

$$DOP^{out} = \frac{\sqrt{S_1^{in} + S_2^{in} + S_3^{in}}}{S_0^{in}} \cdot P_C = DOP^{in} \cdot P_C \tag{7}$$

where S^{in} with a subscript denotes elements of the Stokes vector.

In the experiment we have measured DOP changes at the output of a lithium niobate anisotropic crystal as shown in Fig. 2. As a light source we have used a semiconductor laser diode (λ=661 nm) with the coherence length ΔL=1.3 mm lasing at a single mode with the Lorentzian shape of a spectrum.

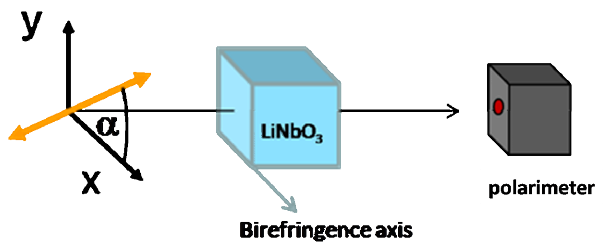

Fig.2. Setup for measuring DOP changes for linearly polarized light

To measure DOP we used high precision Stokes polarimeter Thorlabs PAX-5710. The experimental data were compared with the theoretical curve obtained from Eq. (5) and presented in Fig. 3.

For azimuth α defined as $arctg\left(E_{0x}/E_{0y}\right)$ equal to 0° and 90° electric field vector oscillates along the birefringence axis and DOP is maximal. In the case of α = 45° both orthogonal components of an electric field are excited and DOP reaches its minimum according to Eq. (5).

The coherence length of the laser diode used in the experiment was calculated according to a new non-interferometric method based on the depolarization effect. The coherence length for a laser diode with the Lorentzian spectrum can be derived from equation (5) assuming E_{0x} = E_{0y} (azimuth 45° as in our experiment) [10].

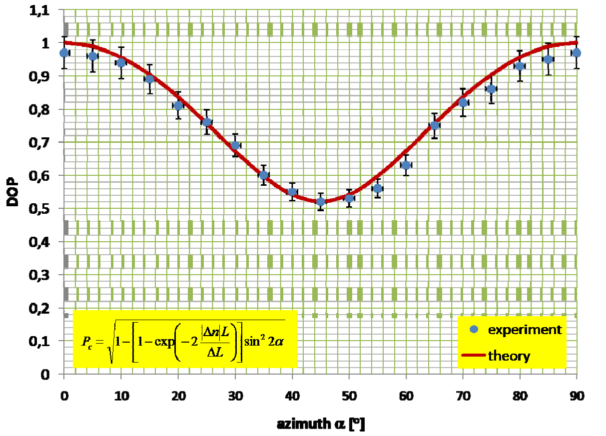

Fig.3. DOP changes in the function of azimuth between the polarization plane and birefringence axis

$$\Delta L = -\frac{|\Delta n| l}{\ln(P_{L_{min}})} \tag{8}$$

In the case of a circular state of polarization of light the E_{0x}/E_{0y} ratio will be constant and DOP should be minimal and no changes should be observed during azimuth changes. To confirm it we have measured DOP changes for circularly polarized light. Circular SOP was generated by the use of a quarter-wave plate in front of a birefringent crystal. A lithium niobate crystal was rotated in order to change the angle β between the birefringent axis and X axis. The presented setup can be described by the Mueller-Stokes matrix equation:

$$\left[\mathbf{S}^{out}\right] = \left[\mathbf{D}_{LiNbO3}\right] \cdot \left[\mathbf{M}(\beta)_{LiNbo3}\right]\left[\mathbf{M}_{QWP}\right] \cdot \left[\mathbf{S}^{in}_{45°}\right] \tag{9}$$

where $[M_{QWP}]$ denotes for the Mueller matrix of a quarter-wave plate and is defined as:

$$\left[\mathbf{M}_{QWP}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{10}$$

$[M(\beta)_{LiNbO3}]$ is the Mueller matrix with rotation angle β of a lithium niobate crystal and defined as:

$$\left[\mathbf{M}(\beta)_{LiNbO3}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\beta) + \sin^2(2\beta)\cos(\delta) & (1-\cos(\delta))\sin(2\beta)\cos(2\beta) & \sin(2\beta)\sin(\delta) \\ 0 & (1-\cos(\delta))\sin(2\beta)\cos(2\beta) & \sin^2(2\beta) + \cos^2(2\beta)\cos(\delta) & \cos(2\beta)\sin(\delta) \\ 0 & \sin(2\beta)\sin(\delta) & -\cos(2\beta)\sin(\delta) & \cos(\delta) \end{bmatrix} \tag{11}$$

and $[D_{LiNbO3}]$ is the depolarization matrix of the crystal.

For Eq. 9 the Stokes vector at the output of a birefringent crystal is:

$$\left[S^{out}\right]=\begin{bmatrix}1\\ P_L\sin(2\beta)\sin(\delta)\\ P_L\cos(2\beta)\sin(\delta)\\ P_L\cos(\delta)\end{bmatrix} \tag{12}$$

The DOP for a light beam with the following Stokes vector is:

$$DOP^{out}=\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}=$$
$$\frac{\sqrt{P_L^2\sin^2(2\beta)\sin^2(\delta)+P_L^2\cos^2(2\beta)\sin^2(\delta)+P_L^2\cos^2(\delta)}}{1}=P_L \tag{13}$$

It means that for circularly polarized light, DOP reaches its minimum and is independent of the azimuth between the birefringent axis and X axis. The experimental setup is shown in Fig. 4.

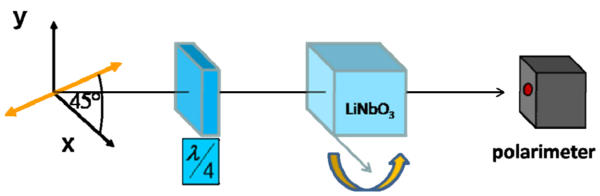

Fig. 4. Setup for measuring DOP changes for circularly polarized light

As expected, the DOP value reached its minimal and constant value with rotation of the crystal. When the crystal is rotated, SOP is changing, but the DOP value remains constant.

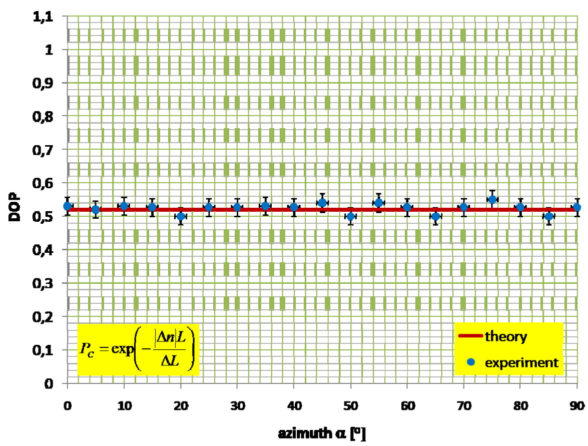

Fig.5. DOP changes as a function of azimuth of the crystal

It means that for any state of polarization of a partially coherent light beam, the DOP value changes dependent not only on birefringence properties of the medium but also on E_x/E_y ratio of a light beam. It should be also added that the depolarization metrix $[D_C]$ introduced 5 years ago [2] for linearly polarized light is suitable for circularly polarized light depolarization analysis as well.

The research leading to this paper was supported by the Polish Ministry of Science under the grant No. 4067/B/T02/2008/34 and by the European Union in the framework of the European Social Fund through the Warsaw University of Technology Development Programme.

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