Bistable solitons in colloidal media

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Abstract—We study nonlinear light propagation in colloidal suspensions of dielectric nanoparticles within the hard-sphere interaction approximation. We analyze the existence and properties of self-trapped beams (spatial optical solitons) in such media and demonstrate the existence of a bistability regime in the one-dimensional case. We find however that the lower stable branch disappears in the two-dimensional model.

Spatial optical solitons are formed when a light-mediated change of the refractive index induces an effective lensing effect that balances diffraction of the laser beam [1]. When an optical beam passes through a colloidal medium composed of a liquid suspension of dielectric nanoparticles, the optical gradient force acts against particle diffusion, increasing the refractive index in regions of higher light intensity. The corresponding local change of the refractive index is of the self-focusing type, and it allows for creation of spatial optical solitons in the form of self-trapped optical beams, as was demonstrated in both theoretical and experimental studies [2–5]. Recently, it was shown theoretically [5] that the optical response of a colloidal medium in the hard-sphere approximation can lead to optical bistability and the existence of two stable soliton solutions for the same beam power, i.e., soliton bistability of the first kind [6,7]. This opens novel opportunities for the control of soliton beams via their collisions and switching [8].

In this letter we review results of our recent studies on solitons in colloidal media and show that the bistability phenomenon, predicted for systems confined in one transverse direction such as planar waveguides or surface waves [2], is absent in the case of bulk colloidal medium, where light can propagate freely in both transverse directions.

The model of nonlinear laser beam propagation in a colloidal suspension of dielectric hard spheres was described in [8]. We assume that the refractive index of colloidal particles, n_p, is slightly higher than the background index n_b and that the particle diameter is much smaller than the laser wavelength in the background medium, $d \ll \lambda_0/n_b$ (Rayleigh regime). We assume that the dielectric colloidal particles interact with each other through a hard-sphere potential. In the steady state the colloidal particles satisfy the Maxwellian velocity distribution, which follows from the phase-space density in the canonical ensemble $\rho \sim \exp(-E/k_B T)$. The pressure exerted by colloidal particles can be obtained from the equation of state in analogy with the hard-sphere gas [9]:

$$\frac{\beta p}{\rho} = Z(\eta), \tag{1}$$

where $\beta = 1/k_B T$, p is the pressure, ρ is the colloidal particle density, $Z(\eta)$ is the compressibility, and $\eta = \rho/\rho_0$ is the packing fraction. In the case of ideal gas, we have Z=1. For a hard-sphere gas, the Carnahan-Starling formula $Z = (1+\eta+\eta^2-\eta^3)/(1-\eta)^3$ gives a very good approximation up to the fluid-solid transition at $\eta \approx 0.5$ [9]. This phenomenological formula is in agreement with exact perturbation theory calculations as well as molecular-dynamics simulations.

In the presence of a slowly varying external potential, such as that induced by the presence of optical beam, the particle velocity distribution is locally Maxwellian. The gradient of the density $\rho(r)$ is assumed to be locally parallel to $\hat{x}$, and we consider a small box of volume $dV = dxdS$, with length dx and normal surface dS. The difference in pressure exerted on the right and left surfaces, dp, gives rise to an effective force acting on the colloidal particles, F_{int}. It is equal to the external force that is necessary to sustain the density gradient, and $dp = -F_{int}/dS = -f_{int}\rho dV/dS = -f_{int}\rho dx$, where f_{int} is the average force acting on a single particle. Using (1), we get $d(\rho Z)/dx = -f_{int}\rho\beta$. The particle current density is equal to

$$\vec{j} = \rho\mu(\vec{f}_{ex} + f_{int}) = \rho\mu\vec{f}_{ex} - D\vec{\nabla}(\rho Z)\;, \tag{2}$$

where μ is the particle mobility, $D = \mu/\beta$ is the diffusion constant, and f_{ex} corresponds to the external optical gradient force.

For steady-state solutions, equation (2) can be solved analytically to give electric field envelope u:

$$|u|^2 = g(\eta) - g(\eta_0)\;, \tag{3}$$

where η_0 is the background colloidal packing fraction, and $g(\eta)=(3-\eta)/(1-\eta)^3+\ln(\eta)$ [8].

The typical dependence $\eta(|u|^2)$ in this case is shown in Fig. 1. In the low-intensity limit, the nonlinear index change is Kerr-like (proportional to intensity). For higher intensities, it is well described by the exponential model of [3,4]. Finally, for higher densities the particle hard-sphere interactions become significant and the nonlinearity saturates as the exponential model breaks down.

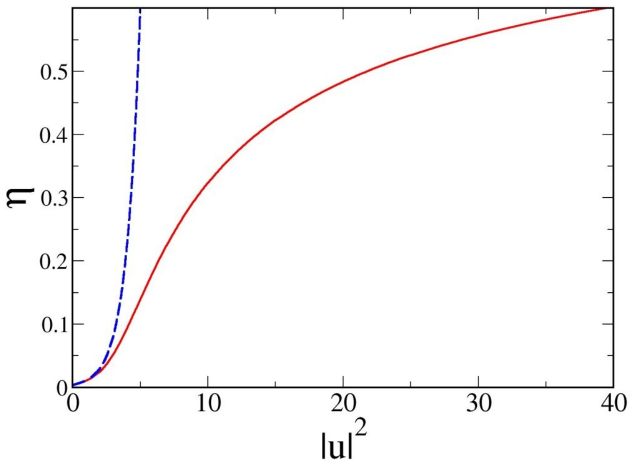

Fig. 1. Packing fraction η of colloidal particles vs the light intensity (solid line). The dashed line shows the dependence of the exponential model.

The light propagation equation can be derived from Helmholtz equation under the slowly varying envelope approximation [8]. In renormalized spatial coordinates, we obtain a generalized Nonlinear Schrödinger Equation (NLS)

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\nabla_{\perp}^2 u+(\eta-\eta_0)u+i\frac{\Gamma}{2}\eta u=0, \tag{4}$$

where the renormalized damping coefficient due to Rayleigh scattering is

$$\Gamma=\frac{2}{3}\pi^3\sqrt{1+3\delta\eta_0}\left(\frac{d}{\lambda}\right)^3\delta\ , \tag{5}$$

and $\delta=(m^2-1)/(m^2+2)$ with $m=n_p/n_b$. From (5), we conclude that the effect of scattering losses depends strongly on the ratio of the particle size to the laser wavelength. In the following, we will ignore the effect of damping in accordance with the assumption $d\ll\lambda_0$.

First, we review results on one-dimensional spatial solitons when $u = u(x,z)$. We look for localized solutions of (4) in the form $u=A(x)\exp(i\kappa z)$. In Fig. 2(a) we show the dependence of the soliton width $W=3\int|x||u|^2dx$ and power $P=\int|u|^2dx$ versus the propagation constant κ . The two stable branches with a branch with a negative slope [6]. Bistable solutions exist within the power range $P\approx33$–51. These solitons fulfil all the three stability conditions required for robustness during collisions [10]. Examples of bistable soliton profiles for P=40 are presented in Figs. 2(b) and 2(c). The width of the soliton from the lower branch is approximately 20 times larger than the width of the soliton from the upper branch carrying the same beam power. Therefore, these solitons can be easily distinguished in experiment by measuring their width. For the experimental parameters λ=1064 nm, particle diameter d=30 nm, n_p=1.56, and n_b=1.33 (polystyrene beads in water), the total soliton beam power is about 2 W and the peak intensity of the soliton from the upper branch reaches 300 MW/cm².

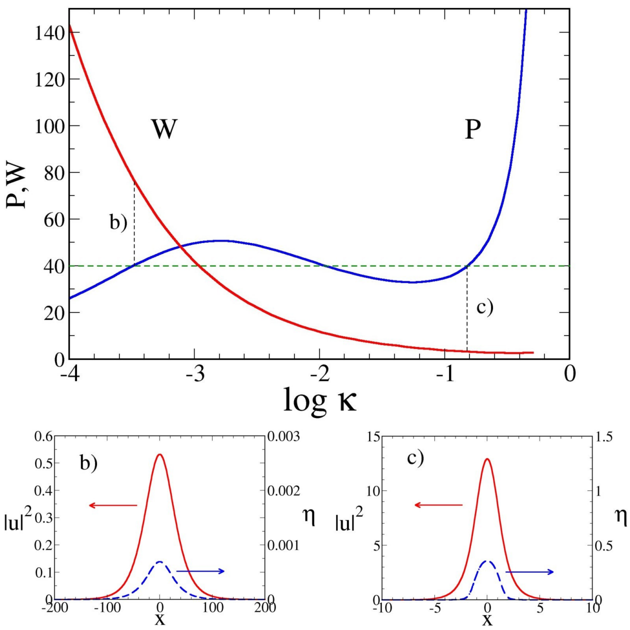

Fig. 2. (a) Soliton power and width vs the propagation constant κ in the one-dimensional case for $\eta_0=10^{-3}$. Bottom panels show the soliton intensity profile (solid) and colloidal particle packing fraction (dashed) for bistable solitons carrying power P=40 from (b) the lower stable branch and (c) the upper stable branch. Notice the difference in the width scale.

In Fig. 3 we present one of the possible scenarios of soliton collisions. The soliton of the lower branch is triggered to switch to a soliton of the upper branch. In a sharp contrast to all previously considered types of interaction between optical solitons [1,11], collisions of solitons belonging to different branches are phase independent and always repulsive. Presently, it is commonly believed that the incoherent interaction of solitons results always in their attraction, but not repulsion [1,11]. In fact, each of the solitons "feels" the other one as an effective attractive potential; hence, the soliton coalescence or passing can be anticipated. However, a soliton reflection from a narrow, deep, attractive potential wells or defects has been already reported for different systems [12,13]. It was shown that if the soliton velocity falls below a certain threshold, a repulsive interaction between the soliton and an attractive potential can occur.

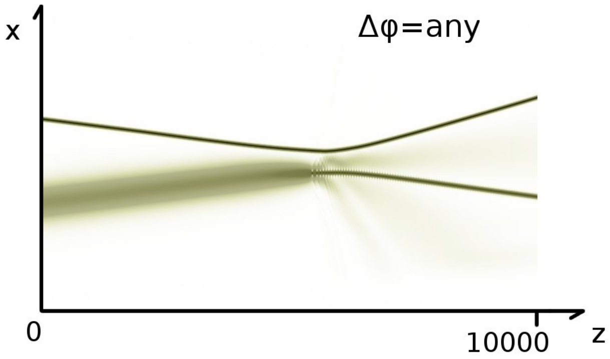

Fig. 3. Switching from the lower to the upper branch triggered by collision with another soliton for soliton power P=50 and $k_0 = 0.01$. Powers of both solitons are equal to P.

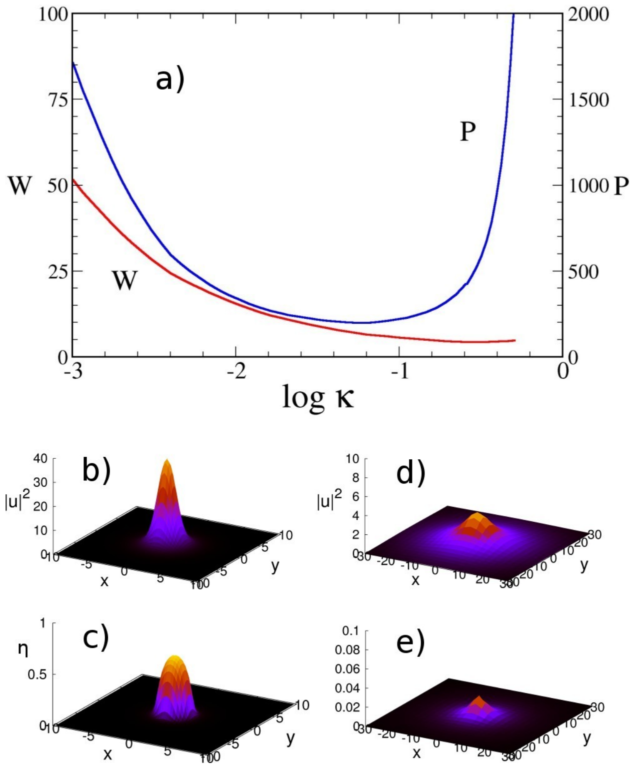

Fig. 4. (a) Soliton power and width vs the propagation constant κ in the two-dimensional case for $\eta_0 = 10^{-3}$. The second stable branch, for low values of κ, is absent.(b),(c) Profile of the light intensity (b) and colloidal packing fraction (c) for the stable soliton with P=300 and $\kappa=0.18$. (d), (e) The same for the unstable soliton with P=1700 and $\kappa=0.001$

In the case of two-dimensional model we have $u = u(x,y,z)$ and the transverse Laplacian in equation (4) contains derivatives with respect to both x and y. The steady-state soliton solutions are presented in Fig. 4 which depicts dependence of soliton width (and power) on the propagation constant κ, and two-dimensional profiles of the solitons. In contrast to the one-dimensional case, there is now only one stable soliton branch with positive slope, for high values of the propagation constant κ. The lower branch disappears, which is a consequence of the fact that two-dimensional solitons are unstable against collapse in the Kerr NLS model [14]. As we saw before, the nonlinear refractive index change in the current model is Kerr-like (proportional to intensity) for low beam intensities. Therefore, one can expect that broad solitons with low light intensity, like those existing at low κ, see Fig. 4(d), will be unstable.

In conclusion, we reviewed theoretical results on soliton bistability in colloidal media in the hard-sphere interaction approximation. We demonstrated that the phenomenon of bistability, present in the one-dimensional geometry, is not present in the two-dimensional model due to instability in the Kerr regime and then disapp-earance of the lower branch of stable solitons.

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