Controlling group velocity via an external magnetic field in a degenerated three-level lambda-type atomic system

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Abstract—The analytical expression for the group index in a degenerated three-level lambda-type atomic system is derived as a function of the parameters of laser fields and an external magnetic field. The influence of an external magnetic field on the group index is investigated. It is shown that by changing the magnitude or sign of an external magnetic field, as well as the transparency window with normal dispersion switches to enhanced absorption with anomalous dispersion at the line center, light propagation can be converted between subluminal and superluminal modes.

Today, group velocity control of a light pulse in a dispersive medium has attracted considerable attention due to its potential applications in many fields such as quantum memories, high-speed optical switches, optical communications, and quantum information processing [1]. In general, group velocity can be controlled via dispersion of the atomic medium.

In recent years, it has been demonstrated that quantum interference and atomic coherence lead to interesting phenomena in quantum optics such as electromagnetically induced transparency (EIT) [2], electromagnetically induced absorption (EIA) [3], and so on. The EIT can deliver a normal dispersive medium with very steep dispersion [4], while the EIA can create an anomalous dispersive medium [5]. Under the EIT and EIA conditions, several theoretical and experimental studies of subluminal [6÷7] and superluminal [8÷10] propagations of light have been done. Especially, for EIT materials, one can easily control light propagation between subluminal and superluminal modes by adjusting the intensity, frequency, polarization, and phase of laser beams [11÷15]. Recently, many investigations have focused on the utilization of an external magnetic field to control the EIT effect [16÷18] and light propagation [19÷20].

In this work, we suggest using an external magnetic field as a "knob" to control the group velocity between the subluminal and superluminal values in a degenerated three-level lambda-type atomic medium. An analytical expression for the group index is found as a function of laser parameters and external magnetic field. The influence of a magnetic field on the group index is investigated. It is shown that by changing the magnitude or the sign of a magnetic field, the group velocity can switch between subluminal and superluminal values.

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The degenerated three-level atomic system in an external magnetic field is represented in Fig. 1(a). A weak probe field E_p with angular frequency ω_p drives the transition $|1\rangle \leftrightarrow |2\rangle$. A strong coupling field E_c with angular frequency ω_c couples the transition $|2\rangle \leftrightarrow |3\rangle$. Here, the probe light is the left-circularly polarization σ^-, while the coupling light is the right-circularly polarization σ^+. The external magnetic field ($\vec{B}$) is arranged so that its direction is parallel to that of the propagation direction of the probe and coupling beams. This is to remove the degeneracy among the ground-state sublevels $|1\rangle$ and $|3\rangle$ via the Zeeman effect as shown in Fig. 1(b). The Zeeman shift of the $|1\rangle$ and $|3\rangle$ levels is determined by $\hbar\Delta_B = \mu_B m_F g_F B$, where μ_B is the Bohr magneton, g_F is the Landé factor, and $m_F = \pm 1$ is the magnetic quantum number. We use γ_{21} and γ_{23} to denote respectively the decay rates from the $|2\rangle$ state to the $|1\rangle$ and $|3\rangle$ states, while γ_{31} is the relaxation rate of atomic coherence between the $|1\rangle$ and $|3\rangle$ states by collisions of atoms.

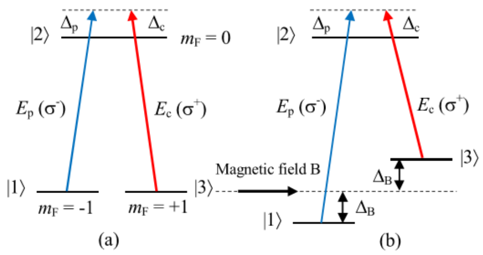

Fig. 1. (a) The three-level lambda system without an external magnetic field. (b) The three-level lambda system with an external magnetic field: the $|1\rangle$ state is lowered, while the $|3\rangle$ state is lifted by the same amount Δ_B equal to the Zeeman shift.

The evolution of the system in laser fields is described by the density matrix equations as follows:

$$\dot{\rho}_{11} = \gamma_{31}(\rho_{33}-\rho_{11}) + \gamma_{21}\rho_{22} - \frac{i}{2}\Omega_p\rho_{21} + \frac{i}{2}\Omega_p\rho_{12}, \quad (1a)$$

$$\dot{\rho}_{33} = \gamma_{31}(\rho_{11}-\rho_{33}) + \gamma_{23}\rho_{22} + \frac{i}{2}\Omega_c\rho_{32} - \frac{i}{2}\Omega_c\rho_{23}, \quad (1b)$$

$$\dot{\rho}_{22} = -(\gamma_{23}+\gamma_{21})\rho_{22} - \frac{i}{2}\Omega_p\rho_{12} + \frac{i}{2}\Omega_p\rho_{21} - \frac{i}{2}\Omega_c\rho_{32} + \frac{i}{2}\Omega_c\rho_{23}, \quad (1c)$$

$$\dot{\rho}_{21} = -[\gamma - i(\Delta_p - \Delta_B)]\rho_{21} + \frac{i}{2}\Omega_p(\rho_{22}-\rho_{11}) - \frac{i}{2}\Omega_c\rho_{31}, \quad (1d)$$

$$\dot{\rho}_{23} = -[\gamma - i(\Delta_c + \Delta_B)]\rho_{23} + \frac{i}{2}\Omega_c(\rho_{22} - \rho_{33}) - \frac{i}{2}\Omega_p\rho_{13}\,, \tag{1e}$$

$$\dot{\rho}_{31} = -[\gamma_{31} - i(\Delta_p - \Delta_c - 2\Delta_B)]\rho_{31} + \frac{i}{2}\Omega_p\rho_{32} - \frac{i}{2}\Omega_c\rho_{21}\,, \tag{1f}$$

$$\rho_{nm} = \rho^*_{mn}\,, \tag{1g}$$

$$\rho_{11} + \rho_{22} + \rho_{33} = 1\,. \tag{1h}$$

where $\gamma = (\gamma_{21} + \gamma_{23} + \gamma_{31})/2$; $\Delta_p = \omega_p - \omega_{21}$ and $\Delta_c = \omega_c - \omega_{23}$ are respectively the frequency detuning of the probe and coupling fields; the Rabi frequency of the probe and coupling fields are represented by $\Omega_p = d_{21}E_p/\hbar$ and $\Omega_c = d_{23}E_c/\hbar$ with d_{mn} being the electric-dipole moment of the transition $|m\rangle \leftrightarrow |n\rangle$.

By solving the density matrix Eqs. (1) in the steady-state $\partial\rho/\partial t = 0$, we found the solution for ρ_{21} under weak-probe approximation as:

$$\rho_{21} = \frac{\frac{i}{2}\Omega_p(\rho^{(0)}_{22} - \rho^{(0)}_{11})}{\gamma - i(\Delta_p - \Delta_B) + \frac{(\Omega_c/2)^2}{\gamma_{31} - i(\Delta_p - \Delta_c - 2\Delta_B)}} \approx \frac{-i\Omega_p}{4F}\,, \tag{2}$$

where

$$F = \gamma - i(\Delta_p - \Delta_B) + \frac{(\Omega_c/2)^2}{\gamma_{31} - i(\Delta_p - \Delta_c - 2\Delta_B)}\,. \tag{3}$$

and we assumed that the atoms are initially in the ground states $|1\rangle$ and $|3\rangle$ with the same populations, $\rho^{(0)}_{11} \approx \rho^{(0)}_{33} \approx 1/2$, and $\rho^{(0)}_{22} \approx 0$.

The susceptibility χ for the probe beam is proportional to ρ_{21} as follows:

$$\chi = -2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21} \equiv \frac{Nd_{21}}{\varepsilon_0 E_p}\frac{i\Omega_p}{2F}\,. \tag{4}$$

where N is the atomic density and ε_0 is the vacuum permittivity. After extracting the real and imaginary parts of the susceptibility χ, we obtain:

$$\chi = \frac{Nd^2_{21}}{\varepsilon_0\hbar}\left(\frac{A}{A^2 + B^2} + i\frac{B}{A^2 + B^2}\right), \tag{5}$$

with A and B determined by

$$A = -(\Delta_p + \Delta_B) + \frac{(\Delta_p - \Delta_c - 2\Delta_B)}{\gamma^2_{31} + (\Delta_p - \Delta_c - 2\Delta_B)^2}\left(\frac{\Omega_c}{2}\right)^2, \tag{6}$$

$$B = \gamma + \frac{\gamma_{31}}{\gamma^2_{31} + (\Delta_p - \Delta_c - 2\Delta_B)^2}\left(\frac{\Omega_c}{2}\right)^2. \tag{7}$$

Thus, the absorption (α) and dispersion (n_0) coefficients, which are related to the real and imaginary parts of χ, are given by:

$$\alpha = \frac{Nd^2_{21}\omega_p}{2c\varepsilon_0\hbar}\frac{B}{A^2 + B^2}\,, \tag{8}$$

$$n_0 = 1 + \frac{\mathrm{Re}(\chi)}{2} = 1 + \frac{Nd^2_{21}}{2\varepsilon_0\hbar}\frac{A}{A^2 + B^2}\,. \tag{9}$$

The group velocity of the probe light is defined by

$$v_g = \frac{c}{n_g}\,, \tag{10}$$

here c is the speed of light in the vacuum and n_g is the group index which is determined as

$$n_g = n_0 + \omega_p\frac{dn_0}{d\omega_p};\ \omega_p\frac{Nd^2_{21}}{2\varepsilon_0\hbar}\left[\frac{A'(A^2 + B^2) - 2A(AA' + BB')}{(A^2 + B^2)^2}\right]. \tag{11}$$

with A' and B' are respectively the derivatives of A and B over ω_p, we have:

$$A' = -1 + \frac{(\Omega_c/2)^2}{\gamma^2_{31} + (\Delta_p - \Delta_c - 2\Delta_B)^2} - \frac{2(\Delta_p - \Delta_c - 2\Delta_B)^2}{[\gamma^2_{31} + (\Delta_p - \Delta_c - 2\Delta_B)^2]^2}\left(\frac{\Omega_c}{2}\right)^2, \tag{12}$$

$$B' = -\frac{2\gamma_{31}(\Delta_p - \Delta_c - 2\Delta_B)}{[\gamma^2_{31} + (\Delta_p - \Delta_c - 2\Delta_B)^2]^2}\left(\frac{\Omega_c}{2}\right)^2. \tag{13}$$

Now, we use the analytical results to ^{87}Rb atomic medium. The states $|1\rangle$, $|2\rangle$ and $|3\rangle$ are respectively $5S_{1/2}(F = 1, m_F = +1)$, $5P_{1/2}(F = 2, m_F = 0)$, and $5S_{1/2}(F = 2, m_F = -1)$. The atomic density $N = 4.5\times10^{17}$ atoms/m^3, $\gamma_{21} = \gamma_{23} = 5.3$ MHz, $d_{21} = 1.6\times10^{-29}$ C.m. The Landé factor is $g_F = -1/2$, and the Bohr magneton is $\mu_B = 9.27401\times10^{-24}$ JT^{-1}. For simplicity, the parameters in frequency units are normalized in γ. In this way, the magnetic field strength B is also normalized by a constant $\gamma_c = \gamma\hbar/(\mu_B g_F)$. For example, when taking the Zeeman shift $\Delta_B = 0.5\gamma$, then the magnetic field strength $B = \Delta_B\hbar/(\mu_B m_F g_F) = 0.5\gamma_c$.

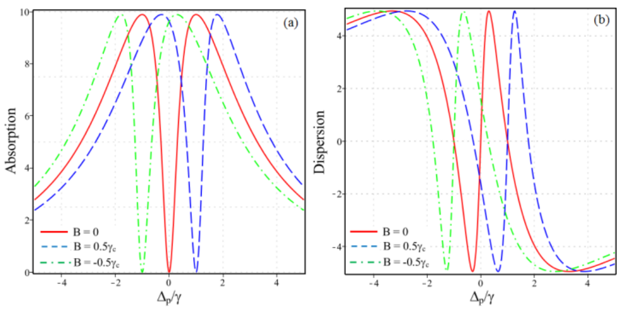

Fig. 2. Variations of absorption (a) and dispersion (b) versus probe frequency detuning when $\Omega_c = 2\gamma$, $\Delta_c = 0$ and B = 0 (solid line), B = $0.5\gamma_c$ (dashed line) and B = $-0.5\gamma_c$ (dash-dotted line).

Figure 2 shows the variations of the absorption (α) and dispersion (n_0) coefficients versus the probe frequency detuning Δ_p for different values of magnetic field B = 0 (solid line), B = 0.5γ (dashed line) and B = -0.5γ (dash-dotted line), and the coupling field parameters $\Omega_c = 2\gamma$ and $\Delta_c = 0$. From Fig. 2(a) we see that when the magnetic field is absent, i.e., B = 0 (which corresponds to $\Delta_B = 0$), the position of the transparency window is localized at the line center of the absorption profile. However, when the external magnetic field is switched on, the transparency window has moved to the left by an amount of $\Delta_p = 1\gamma$ for the case of B = $-0.5\gamma_c$ (which corresponds to $\Delta_B = -0.5\gamma$) and moved to the right by the same amount $\Delta_p = 1\gamma$ for the case of B = $-0.5\gamma_c$ (which corresponds to $\Delta_B = 0.5\gamma$). At the same time, a strong absorption peak appears at resonance frequency. This means that the medium is switched from transparent to absorbed regimes and vice versa.

Along with the transition between EIT and EIA, the dispersion is also switched from normal to abnormal regimes in the resonant region via turn-on/off of the external magnetic field, as shown in Fig. 2(b). Thus, the external magnetic field can be used as a "knob" to convert light propagation between superluminal to subluminal modes, as depicted in Fig. 3. In Fig. 3, we plot the group index versus probe frequency detuning Δ_p for different values of the external magnetic field B = 0 (solid line), B = $0.5\gamma_c$ (dashed line) and B = $-0.5\gamma_c$ (dash-dotted line) while the parameters of coupling field are $\Omega_c = 2\gamma$ and $\Delta_c = 0$. It shows that the positive peak of the group index at $\Delta_p = 0$ when B = 0 has switched to the negative peak when B = $\pm 0.5\gamma_c$. Otherwise, the negative values of the group index at $\Delta_p = \pm 1\gamma$ when B = 0 has switched to the positive peak with B = $0.5\gamma_c$ or B = $-0.5\gamma_c$.

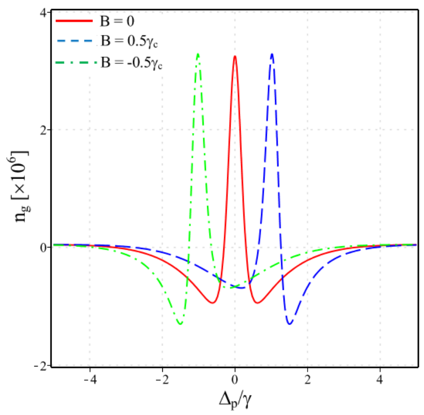

Fig. 3. Variations of the group index versus probe frequency detuning when B = 0 (solid line), B = $0.5\gamma_c$ (dashed line) and B = $-0.5\gamma_c$ (dash-dotted line). Other parameters are $\Omega_c = 2\gamma$ and $\Delta_c = 0$.

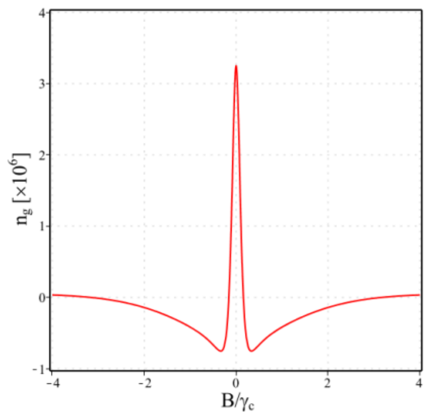

Fig. 4. Variation of the group index via magnetic field when $\Delta_p = 0$, $\Delta_c = 0$ and $\Omega_c = 2\gamma$.

Figure 4 shows the variation of the group index versus the magnetic field at two-photon resonance $\Delta_p = \Delta_c = 0$ and $\Omega_c = 2\gamma$, which corresponds to the positive peak of the group index at $\Delta_p = 0$ when B = 0. From Fig. 4 we can see that both the magnitude and the sign of the group index are varied according to the magnetic field.

We have found the analytical expression for the group index of a degenerated three-level lambda-type atomic medium in the external magnetic field. The effect of the magnetic field on the group index has been investigated. It has been shown that the magnitude and the sign of the group index are changed by the magnetic field. This means that the magnetic field can be used as a "knob" to convert the medium response between electromagnetically induced transparency and electromagnetically induced absorption, and switch light propagation between subluminal and superluminal modes.

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